On the Distinction between Active and Passive Reaction in Grasp Stability Analysis *

Maximilian Haas-Heger[1], Garud Iyengar[2], and Matei Ciocarlie[1]

[1] Department of Mechanical Engineering, Columbia University, New York, NY 10027, USA; m.haas@columbia.edu, matei.ciocarlie@columbia.edu

[2] Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA; garud@ieor.columbia.edu

Abstract. Stability analysis for multi-fingered robotic grasping is often formalized as the problem of answering the following questions: can the hand exert contact forces on the object either without a net resultant wrench (thus loading the contacts while creating purely internal forces) or in order to counterbalance an external disturbance (by applying an equal and opposite wrench). However, some of the most commonly used methods for performing this analysis do not distinguish between active torque generation and passive resistance at the joints (and, in fact, many commonly used methods disregard the actuation mechanism entirely). In this study, we introduce an analysis framework constructed to capture such differences, and present evidence showing that this is an important distinction to make when assessing the stability of a grasp employing some of the most commonly used actuation mechanisms.

1 Introduction

Stability analysis is one of the foundational problems for multi-fingered robotic manipulation. Grasp planning can be posed as a search over the space of possible grasps looking for instances that satisfy a measure of stability. The formulation and characteristics of the stability measure thus play a key role in this search, and, by extension, in any task that begins by planning and executing a grasp.

In turn, multi-fingered grasp stability relies on studying the net resultant wrench imparted by the hand to the grasped object. Ferrari and Canny [8] introduced a very efficient geometric method for determining the total space of possible resultant wrenches as long as each individual contact wrench obeys (linearized) friction constraints. This method answers the simplest form of what we refer to here as the existence problem: given a desired output, are there legal contact wrenches that achieve it, and, if so, how large is their needed magnitude in relation to the output? This approach has been at the foundation of numerous planning algorithms proposed since.

Consider a grasp that scores highly according to the quality metric described above. This means that any desired resultant can be produced by a computable

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set of contact wrenches (of bounded magnitude). In turn, the contact wrenches can be balanced by a set of joint torques, which can also be computed [13]. However, this approach is based on a string of assumptions:

- First, we have assumed that, at any given moment, the control mechanism knows what resultant wrench is needed on the object.
- Second, we have assumed that the joint torques needed to balance this resultant can be actively commanded by the motor outputs.
- Third and finally, we have assumed that the desired motor output torques can be obtained accurately.

In practice, these assumptions do not always hold. The external wrench in need of balancing is difficult to obtain: to account for gravity and inertia, one needs the exact mass properties and overall trajectory of the object, which are not always available; any additional external disturbance will be completely unknown, unless the hand is equipped with tactile sensors. Finally, many commonly used robot hands use highly geared motors unable to provide accurate torque sensing or regulation.

A much simpler approach, applicable to more types of hardware, is to simply select a set of motor commands that generate some level of *preload* on the grasp, and maintain that throughout the task. This method assumes that the chosen motor commands will not only lead to an adequate and stable preload for grasp creation, but also prove suitable for the remainder of the task. A key factor that allows this approach to succeed is *the ability of a grasp to absorb forces that would otherwise unbalance the system without requiring active change of the motor commands.*

Following the arguments above, we believe it is important to not only consider the wrenches the hand can apply actively by means of its actuators, but also the reactions that arise passively. Thus, in this study we are interested in the distinction between *active force generation*, directly resulting from forces applied by a motor, and *passive force resistance*, arising in response to forces external to the contacts or joints. Consider the family of highly geared, non-backdrivable motors: the torque applied at a joint can exceed the value provided by the motor, as long as it arises passively, in response to torques applied by other joints, or to external forces acting on the object. Put another way, joint torques are not always the same as output motor torques, even for direct-drive hands.

From a practical standpoint, a positive answer to the existence problem outlined above is not useful as long as the joint torques necessary for equilibrium will not be obtained given a particular set of commands sent to the motors. We focus on stability from an inverse perspective: given a set of commanded torques to be actively applied to the robot's joints, what is the net effect expected on the grasped object, accounting for passive reactions?

Overall, the main contribution of this paper is a quasi-static grasp stability analysis framework to determine the *passive* response of the hand-object system to applied joint torques and externally applied forces. This method was designed to account for actuation mechanisms such as non-backdrivable or position-controlled motors.

2 Related Work

The problem of force distribution between an actively controlled robotic hand and a rigid object has been considered by a number of authors [14, 1, 10, 16]. A great simplification to grasp analysis is the assumption that any contact force can be applied by commanding the joint motors accordingly. This assumption neglects the deficiency of the kinematics of many commonly used robotic hands in creating arbitrary contact forces. The idea that the analysis of a grasp must include not only the geometry of the grasp but also the kinematics of the hand is central to this paper.

Bicchi [2] showed that for a kinematically deficient hand only a subset of the internal grasping forces is actively controllable. Using a quasi-static model, the subspace of internal forces was decomposed into subspaces of active and passive internal forces. Making use of this decomposition Bicchi proposed a method to pose the problem of optimal distribution of contact forces with respect to power consumption and given an externally applied wrench as a quadratic program [3]. He proposed a definition of force-closure that makes further use of this decomposition and developed a quantitative grasp quality metric that reflects on how close a grasp is to losing force-closure under this definition [4].

There have been rigid body approaches to the analysis of active and passive grasp forces. Yoshikawa [15] studied the concept of active and passive closures and the conditions for these to hold. Melchiorri [11] decomposed contact forces into four subspaces using a rigid body approach. Burdick and Rimon [5] formally defined four subspaces of contact forces and gave physically meaningful interpretations. They analyzed active forces in terms of the injectivity of the transposed hand Jacobian matrix. They note that the rigid body modeling approach is a limitation, as a compliance model is required to draw conclusions on the stability of a grasp.

An important distinction between our work and that of the above authors lies in the definition of what qualifies as a "passive" contact force. In addition to contact forces that lie in the null space of the transposed hand Jacobian, we consider contact forces arising from joints being loaded passively (due to the non-backdrivability of highly geared motors) and not arising from the commanded joint torque as passive. Furthermore, we define preload forces as the internal forces that arise from selecting a set of motor commands that achieve a grasp in stable equilibrium, previous to the application of any external wrench.

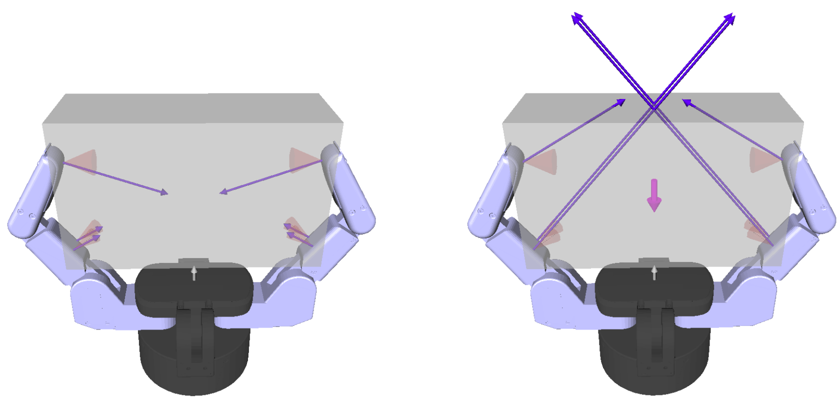
3 Problem Statement

Consider a grasp establishing multiple contacts between the robot hand and the grasped object. We denote the vector of contact wrenches by $\boldsymbol{c}$. The grasp map matrix $\boldsymbol{G}$ relates contact wrenches to the net resultant wrench $\boldsymbol{w}$, while the transpose of the grasp Jacobian $\boldsymbol{J}$ relates contact wrenches to joint torques $\boldsymbol{\tau}$:

$$\boldsymbol{G}\boldsymbol{c} = \boldsymbol{w} \tag{1}$$

$$\boldsymbol{J}^T\boldsymbol{c} = \boldsymbol{\tau} \tag{2}$$

The classical approach: In combination with a contact constraint model, the relatively simple system of Eqs. (1)&(2) expresses the static equilibrium constraints for the grasp. However, for rigid bodies, the problem of computing the exact force distribution across contacts in response to an applied wrench is statically indeterminate. Thus, previous studies such as those of Bicchi [2–4] make use of a linear compliance matrix that characterizes the elastic elements in a grasp and solves the indeterminacy. For a comprehensive study on how to compute such a compliance matrix see the work by Cutkosky and Kao [7]. A compliance matrix allows us to consider the force distribution across contacts as the sum of a particular and a homogeneous solution. The contact forces $\boldsymbol{c_0}$ create purely internal forces and hold the object in equilibium. This is the homogeneous solution and as noted in the Introduction, it can be of great importance to grasp stability. The contact forces $\boldsymbol{c_p}$ associated with the application of an external wrench $\boldsymbol{w}$ are considered the particular solution. Bicchi formulates a *force distribution problem* [3] given by $\boldsymbol{c} = \boldsymbol{c_p} + \boldsymbol{c_0} = \boldsymbol{G_K^R w} + \boldsymbol{c_0}$ where $\boldsymbol{G_K^R}$ is the $\boldsymbol{K}$-weighted pseudoinverse of the grasp map matrix $\boldsymbol{G}$. $\boldsymbol{K}$ is the stiffness matrix of the grasp and is given by the inverse of the grasp compliance matrix.



(a) Homogeneous (b) Homogeneous + Particular

Fig. 1: Illustration of the shortcomings of a linear compliance model. The homogeneous solution was obtained using the algorithm presented in this paper. The particular solution was computed using the linear compliance approach [3]. Contacts have unity stiffness in the normal direction. The stiffness in the frictional direction was set equal to the coefficient of friction. The joints are assumed infinitely stiff. Friction cones are shown in red and corresponding contact forces are shown as blue arrows. The violet arrow denotes the applied force.

Given the subspace of controllable internal forces (see [2]), the particular solution computed in this way can be used to compute a homogeneous solution such that $\boldsymbol{c}$ satisfies all contact constraints. Using Eq. (2) the required equilibrium joint torques that satisfy this system τ_{eq} can then be calculated.

Limitations of this approach: The use of a linear compliance matrix is an important limitation, as it assumes a linear stiffness of the contacts and the joints. However, a contact force may only "push", it cannot "pull" and hence contact forces behave in a nonlinear fashion. Furthermore, a linear compliance model disregards the nonlinearity of frictional forces obeying Coulombs law of friction. We consider contacts of the *point contact with friction* type, which means that the contact force must lie within its friction cone. A linear compliance model has no notion of this friction constraint and thus cannot distribute forces accordingly once the frictional component of a contact force reaches its limit.

To illustrate this issue, consider Fig. 1. Fig. 1a shows a homogeneous solution to a force distribution problem. Fig. 1b shows the sum of the homogeneous and particular solutions when an external wrench pushing the object towards the palm of the robot is applied. Applying a downward force has caused the contact forces on the distal links to violate the friction constraint (they lie outside their respective cones), perhaps leading us to believe that we have to increase the internal forces in the grasp in order to resist the applied wrench. In reality, however, the contacts on the distal links will only apply as much frictional force as they may and more force will be distributed to the contacts on the proximal links. Indeed, experimental results indicate that this grasp withstands arbitrary downward forces applied to the object even in the absence of internal forces.

Furthermore, the compliance of the joints in a robotic hand may be non-linear. Consider the distinction between the equilibrium joint torques τ_{eq}, and those commanded by the motors $\boldsymbol{\tau}_c$. The simplest approach would be to simply set $\boldsymbol{\tau}_c = \boldsymbol{\tau}_{eq}$. However, this approach is subject to the assumptions outlined in the Introduction, requiring that $\boldsymbol{w}$ be known, that the hand have complete authority over all needed joint degrees of freedom (in other words, that one can obtain $\boldsymbol{\tau}_{eq}$ by commanding $\boldsymbol{\tau}_c$), and that $\boldsymbol{\tau}_c$ can be accurately produced by the motors. A simpler approach, and more commonly used in practice, is to command a $\boldsymbol{\tau}_c$, and rely on $\boldsymbol{\tau}_{eq} \neq \boldsymbol{\tau}_c$ arising through passive reactions. For the large family of hands powered by geared, non-backdrivable motors, at any joint i the resulting torque τ^i can exceed the commanded value τ_c^i, but only passively, in response to the torques $\tau^j,\ j \neq i$ and the wrench $\boldsymbol{w}$. We state our problem as follows: given commanded torques $\boldsymbol{\tau}_c$, can the system find quasi-static equilibrium for a net resultant object wrench $\boldsymbol{w}$, assuming passive reaction effects at the joints? In the next section, we present our method for computing an answer, then apply it to the problem of assessing stability for a range of grasps and wrenches.

4 Grasp Model

Due to the above limitations of linear models, we propose a model that accounts for non-linear effects due to the behavior of contact forces and non-backdrivable actuators. To capture the passive behavior of the system in response to external disturbance, we (as others before [9, 2–4]) rely on computing virtual object movements in conjunction with virtual springs placed at the contacts between the rigid object and the hand mechanism. Unlike previous work however, we attempt to also capture effects that are non-linear w.r.t. virtual object movement.

In order to express (linearized) friction constraints at each contact, contact forces are expressed as linear combinations of the edges that define contact friction pyramids, and restricted to lie inside the pyramid:

$$\boldsymbol{D}\boldsymbol{\beta} = \boldsymbol{c} \tag{3}$$

$$\boldsymbol{F}\boldsymbol{\beta} \leq 0 \tag{4}$$

Details on the construction of the linear force expression matrix $\boldsymbol{D}$ and the friction matrix $\boldsymbol{F}$ can be found in the work of Miller and Christensen [12]. Note that while the friction model is linear, in contrast to the linear compliance model discussed in the previous section the frictional forces are not linearly related to virtual object movements. In fact, friction forces are not related to virtual object motion at all. Instead, we propose an algorithm that searches for equilibrium contact forces everywhere inside the friction cones. In this study we use the Point Contact With Friction model, however, the formulation is general enough for other linearized models, such as the Soft Finger Contact [6]. For our purposes, the vector $\boldsymbol{\beta}$, denoting force amplitudes along the edges of the friction pyramids, completely defines contact forces.

Assuming virtual springs placed at the contacts, the normal force at a contact l is determined by the virtual relative motion between the object and the robot hand at that contact in the direction of the contact normal. This can be expressed in terms of virtual object displacements $\boldsymbol{x}$ and virtual joint movements $\boldsymbol{q}$. Matrix $\boldsymbol{N}^l$ selects only the normal component of this relative displacement at contact l. Matrix $\boldsymbol{S}^l$ selects only the normal force at contact l out of vector $\boldsymbol{\beta}$. For simplicity, we choose unity stiffness for the virtual contact spring ($k = 1$).

$$\boldsymbol{S}^l\boldsymbol{\beta} = k\boldsymbol{N}^l(\boldsymbol{G}^T\boldsymbol{x} - \boldsymbol{J}\boldsymbol{q}) \tag{5}$$

However, a contact may only apply positive force (it may only push, not pull). Hence, if the virtual object and joint movements are such that the virtual spring extends from its reference position, the contact force must be zero.

$$\boldsymbol{S}^l\boldsymbol{\beta} \geq 0 \tag{6}$$

$$\boldsymbol{S}^l\boldsymbol{\beta} - k\boldsymbol{N}^l(\boldsymbol{G}^T\boldsymbol{x} - \boldsymbol{J}\boldsymbol{q}) \geq 0 \tag{7}$$

$$\boldsymbol{S}^l\boldsymbol{\beta} \cdot (\boldsymbol{S}^l\boldsymbol{\beta} - k\boldsymbol{N}^l(\boldsymbol{G}^T\boldsymbol{x} - \boldsymbol{J}\boldsymbol{q})) = 0 \tag{8}$$

This is a non-convex quadratic constraint and as such not readily solvable. (Note that if re-posed as a Linear Complementarity Problem it produces a non positive-definite matrix relating the vectors of unknowns). However, the same problem can be posed as a set of linear inequality constraints instead, which can be solved by a mixed-integer programming solver.

$$\boldsymbol{S}^l\boldsymbol{\beta} \geq 0 \tag{9}$$

$$\boldsymbol{S}^l\boldsymbol{\beta} \leq k_1 \cdot y_1^l \tag{10}$$

$$\boldsymbol{S}^l\boldsymbol{\beta} - k\boldsymbol{N}^l(\boldsymbol{G}^T\boldsymbol{x} - \boldsymbol{J}\boldsymbol{q}) \geq 0 \tag{11}$$

$$\boldsymbol{S}^l\boldsymbol{\beta} - k\boldsymbol{N}^l(\boldsymbol{G}^T\boldsymbol{x} - \boldsymbol{J}\boldsymbol{q}) \leq k_2 \cdot (1 - y_1^l) \tag{12}$$

Each contact l is assigned a binary variable y_1^l determining if the normal force at that contact is equal to the force in the virtual spring (for positive spring forces) or zero. Constants k_1 and k_2 are virtual limits that have to be carefully chosen such that the magnitude of the expressions on the left-hand side never exceed them. However, they should not be chosen too large or the problem may become numerically ill-conditioned.

The mechanics of the hand place constraints on the virtual motion of the joints. To clarify this point, consider the equilibrium joint torque τ, at which the system settles, and which may differ from the commanded joint torque τ_c. At any joint i the torque may exceed the commanded value, but only passively. In non-backdrivable hands this means the torque at a joint may only exceed its commanded level if the gearing between the motor and the joint is absorbing the additional torque. In consequence, a joint at which the torque exceeds the commanded torque may not display virtual motion. Defining joint motion, which closes the hand on the object as positive, this constraint can be expressed as another linear complementarity. Similarly to the linear complementarity describing normal contact forces this constraint can be posed as a set of linear inequalities.

$$\tau^i \geq \tau_c^i \tag{13}$$
$$\tau^i \leq \tau_c^i + k_3 \cdot y_2^i \tag{14}$$
$$q^i \leq k_4 \cdot (1 - y_2^i) \tag{15}$$

Each joint is assigned a binary variable y_2^i that determines if the joint may move or is being passively loaded and hence stationary. Similarly to k_1 and k_2 the constants k_3 and k_4 are virtual limits and should be chosen with the same considerations in mind.

5 Solution Method

The computational price we pay for considering these non-linear effects is that virtual object movement is not directly determined by the compliance-weighted inverse of the grasp map matrix; rather, it becomes part of the complex mixed-integer problem we are trying to solve. In general, if a solution exists, there is an infinite number of solutions satisfying the constraints. The introduction of an optimization objective leads to a single solution. A physically well motivated choice of objective might be to minimize the energy stored in the virtual springs. We formulate a *passive response problem* (or PRP) as outlined in Algorithm 1.

In certain circumstances, this formulation proves to be insufficient. The rigid, passively loaded fingers allow an optimization formulation with unconstrained object movement to "wedge" the object between contacts creating large contact forces. This allows the grasp to withstand very large applied wrenches by performing unnatural virtual displacements (see Fig. 2a). To address this, we constrain the object movement such that motion is only allowed in the direction of the unbalanced wrench acting on the object: $\boldsymbol{x} = s\boldsymbol{w}, \quad s \in \mathbb{R}_{\geq 0}$. We replace the objective of the optimization formulation such as to minimize the net resultant wrench $\boldsymbol{r} = \boldsymbol{w} + \boldsymbol{G}^T\boldsymbol{\beta}$ (applied wrench and contact forces). However, under

Algorithm 1

Input: $\boldsymbol{\tau}_c$ - commanded joint torques, $\boldsymbol{w}$ - applied wrench
Output: $\boldsymbol{\beta}$ - equilibrium contact forces
procedure PASSIVE RESPONSE PROBLEM($\boldsymbol{\tau}_c, \boldsymbol{w}$)
    minimize: $(\boldsymbol{S\beta})^T(\boldsymbol{S\beta})$ ▷ energy stored in virtual springs
    subject to:
        Eq. (4)   ▷ friction
        Eqs. (9) − (12)   ▷ virtual springs complementarity
        Eqs. (13) − (15)   ▷ joint complementarity
    return $\boldsymbol{\beta}$
end procedure

this constraint, the solver will generally not be able to completely balance out the wrench in a single step; even after the optimization, some level of unbalanced wrench may remain. To eliminate it, we call the same optimization procedure in an iterative fashion, where, at step i we allow additional object movement in the direction of the unbalanced wrench $\boldsymbol{r}_{i-1}$ remaining from the previous call:

$$\boldsymbol{x}_i = \boldsymbol{x}_{i-1} + s\boldsymbol{r}_{i-1}, \quad s \in \mathbb{R}_{\geq 0} \tag{16}$$

After each iteration, we check for convergence by comparing the incremental improvement to a threshold ϵ. If the objective has converged to a sufficiently small net wrench, we deem the grasp to be stable; otherwise, if the objective converges to a larger value, we deem the grasp unstable. Thus, we formulate a *movement constrained passive response problem* as outlined in Algorithm 2 to be solved iteratively as outlined in Algorithm 3.

Algorithm 2

Input: $\boldsymbol{\tau}_c$ - commanded joint torques, $\boldsymbol{w}$ - applied wrench, $\boldsymbol{x}$ - current object displacement, $\boldsymbol{r}$ - current net wrench
Output: $\boldsymbol{\beta}$ - contact forces, $\boldsymbol{x}_{next}$ - next step object displacement, $\boldsymbol{r}_{next}$ - next step net wrench
procedure MOVEMENT CONSTRAINED PRP($\boldsymbol{\tau}_c, \boldsymbol{w}, \boldsymbol{x}, \boldsymbol{r}$)
    minimize: $\boldsymbol{r}_{next}^T\boldsymbol{r}_{next}$ ▷ net wrench
    subject to:
        Eq. (4)   ▷ friction
        Eqs. (9) − (12)   ▷ virtual springs complementarity
        Eqs. (13) − (15)   ▷ joint complementarity
        Eq. (16)   ▷ object movement
    return $\boldsymbol{\beta}$, $\boldsymbol{x}_{next}, \boldsymbol{r}_{next}$
end procedure

The computation time of this process is directly related to the number of iterations required until convergence. A single iteration takes of the order of 10^{-2}

to 10^{-1} seconds, depending on the complexity of the problem. Most problems converge within less than 50 iterations. All computations were performed on a commodity computer with a 2.80GHz Intel Core i7 processor.

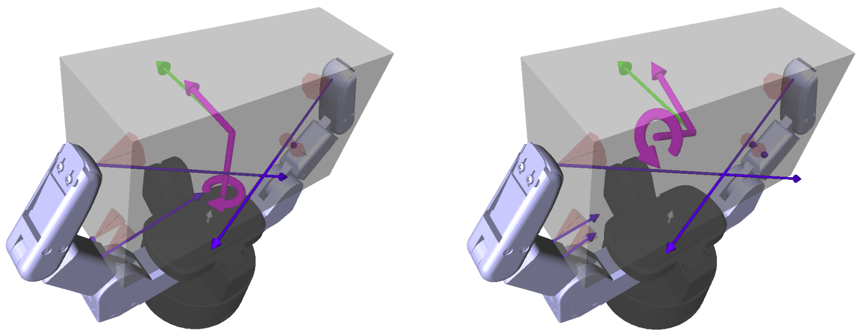
Algorithm 3

```
Input: τ_c - commanded joint torques, w - applied wrench
Output: β - contact forces, r - net resultant
procedure ITERATIVE PASSIVE RESPONSE PROBLEM(τ_c, w)
    x = 0
    r = w
    loop
        (β, x_next, r_next)= Movement Constrained PRP(τ_c, w, x, r)
        if norm(r − r_next) < ϵ then          ▷ Check if system has converged
            break
        end if
        x = x_next
        r = r_next
    end loop
    return β, r
end procedure
```

We use this procedure to answer the question if a grasp, in which the joints are preloaded with a certain commanded torque can resist a given external wrench. In much of the analysis introduced in the next section we are interested in how the maximum external wrench, which a grasp can withstand depends on the direction of application. We approximate the maximum resistible wrench along a single direction using a binary search limited to 20 steps, which requires computation time of the order of tens of seconds. In general, investigating the magnitude of the maximum resistible wrench in every direction involves sampling wrenches in 6 dimensional space. Within our current framework this is prohibitively time consuming and hence we limit ourselves to sampling directions in 2 dimensions and then using the aforementioned binary search to find the maximum resistible wrench along those directions.

6 Analysis and Results

We illustrate the application of our method on two example grasps using the Barrett hand. We show force data collected by replicating the grasp on a real hand and testing resistance to external disturbances. We model the Barrett hand as having all non-backdrivable joints. Our qualitative experience indicates that the finger flexion joints never backdrive, while the spread angle joint backdrives under high load. For simplicity we also do not use the breakaway feature of the hand; our real instance of the hand also does not exhibit this feature. We model the joints as rigidly coupled for motion, and assume that all the torque supplied by each finger motor is applied to the proximal joint.



(a) PRP Solution (Alg. 1) (b) Iterative PRP Solution (Alg. 3)

Fig. 2: Illustration of the shortcomings of directly solving the PRP problem defined above. A force normal to the closing plane of the fingers (illustrated by the green arrow) is applied to the object at its center of mass. The translational and rotational components of the resulting object movement are shown in violet. The PRP algorithm finds a way to wedge the object between the fingers by rotating the object in an unnatural fashion. This enables the solver to find ways to resist arbitrary wrenches. The iterative approach yields the natural finite resistance.

To measure the maximum force that a grasp can resist in a certain direction, we manually apply a load to the grasped object using a Spectra wire in series with a load cell (Futek, FSH00097). In order to apply a pure force, the wire is connected such that the load direction goes through the center of mass of the object. We increase the load until the object starts moving, and take the largest magnitude recorded by the load cell as the largest magnitude of the disturbance the grasp can resist in the given direction.

Case 1. We consider first the case illustrated in Fig. 3. Note that this is the same grasp we used in the Problem Statement section to explain the limitations of the linear compliance model. This grasp can be treated as a 2D problem, considering only forces in the grasp plane, simple enough to be studied analytically, but still complex enough to give rise to interesting interplay between the joints and contacts. Since our simulation and analysis framework is built for 3D problems, we can also study out-of plane forces and in-plane moments.

Consider first the problem of resisting an external force applied to the object CoM and oriented along the Y axis. This simple case already illustrates the difference between active and passive resistance. Resistance against a force oriented along positive Y requires active torque applied at the joints in order to load the contacts and generate friction. The force can be resisted only up to the limit provided by the preload, along with the friction coefficient. If the force is applied along negative Y, resistance happens passively, provided through the contacts on the proximal link. Furthermore, this resistant force does not require any kind of

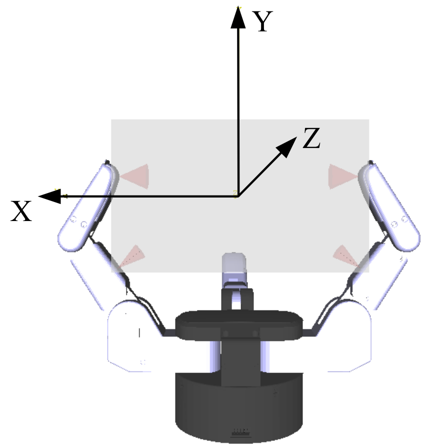


Fig. 3: Grasp example 1.

preload, and is infinite (up to the breaking limit of the hand mechanism, which does not fall within our scope here).

For an external force applied along the X axis, the problem is symmetric between the positive and negative directions. Again, the grasp can provide passive resistance, through a combination of forces on the proximal and distal links. For the more general case of forces applied in XY plane, we again see a combination of active and passive resistance effects. Intuitively, any force with a negative Y component will be fully resisted passively. However, forces with a positive Y component and non-zero X component can require both active and passive responses. Fig. 4 shows the forces that can be resisted in the XY plane, both predicted by our framework and observed by experiment. Note that our formulation predicts the distinction between finite and infinite resistance directions, in contrast to the results obtained using the linear compliance model.

For both real and predicted data, we normalize the force values by dividing with the magnitude of the force obtained along the positive direction of the Y axis (note thus that both predicted and experimental lines cross the Y axis at y=1.0). The plots should therefore be used to compare trends rather than absolute values. We use this normalization to account for the fact that the absolute torque levels that the hand can produce, and which are needed by our formulation in order to predict absolute force levels, can only be estimated and no accurate data is available from the manufacturer. The difficulty in obtaining accurate assessments of generated motor torque generally limits the assessments we can make based on absolute force values.

Moving outside of the grasp plane, Fig. 5 shows predicted and measured resistance to forces in the XZ plane. Again, we notice that some forces can be resisted up to arbitrary magnitudes thanks to passive effects, while others are limited by the actively applied preload.

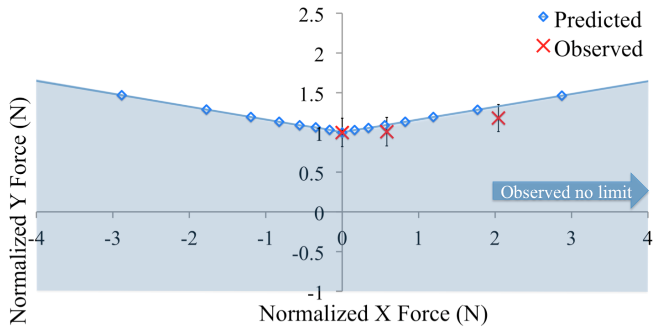


Fig. 4: Normalized forces in the XY plane that can be resisted by grasp example 1: observed by experiment (mean ± one standard deviation) and predicted by our framework (normalized as explained in the text). In all directions falling below the blue line, the prediction framework hit the upper limit of the binary search (arbitrarily set to 1.0e3 N). Hence we deem forces in the shaded area resistible. In the direction denoted by "Observed no limit", the grasp was not disturbed even when hitting the physical limit of the testing setup.

Case 2. One advantage of studying the effect of applied joint torques on grasp stability is that it allows us to observe differences between different ways of preloading the same grasp. For example, in the case of the Barrett hand, choosing at which finger(s) to apply preload torque can change the outcome of the grasp, even though there is no change in the distribution of contacts. We illustrate this approach on the case shown in Fig. 6. We compare the ability of the grasp to resist a disturbance applied along the X axis in the negative direction if either finger 1 or finger 2 apply a preload torque to the grasp. Our formulation predicts that by preloading finger 1 the grasp can resist a disturbance that is 2.37 times higher in magnitude than if preloading finger 2. Experimental data (detailed in Table 1) indicates a ratio for the same disturbance direction of 2.23. The variance in measurements again illustrates the difficulty of verifying such simulation results with experimental data. Nevertheless, experiments confirmed that preloading finger 1 is significantly better for this case.

This result can be explained by the fact that, somewhat counter-intuitively, preloading finger 1 leads to larger contact forces than preloading finger 2, even if the same torque is applied by each motor. Due to the orientation of finger 1, the contact force on finger 1 has a smaller moment arm around the finger flexion axes than is the case for finger 2. Thus, if the same flexion torque is applied in turn at each finger, the contact forces created by finger 1 will be higher. In turn, due to passive reaction, this will lead to higher contact forces on finger 2, even if finger 1 is the one being actively loaded. Finally, these results hold if the spread degree of freedom is rigid and does not backdrive; in fact, preloading finger 1 leads to a much larger passive (reaction) torque on the spread degree of freedom than when preloading finger 2.

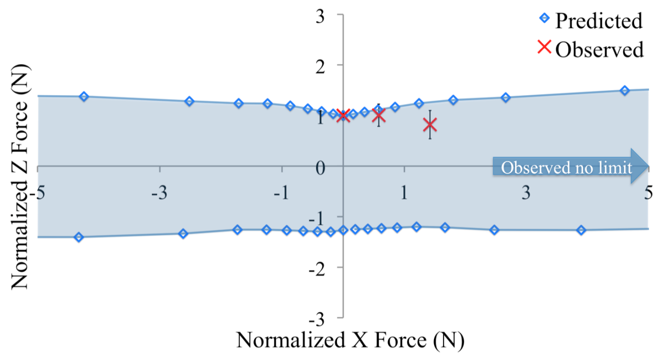


Fig. 5: Normalized forces in the XZ plane that can be resisted by grasp example 1: predicted by our framework, and observed by experiment. In all directions falling between the blue lines (shaded), the prediction framework hit the upper limit of the binary search (arbitrarily set to 1.0e3 N). In the direction denoted by "Observed no limit", the grasp was not disturbed even when hitting the physical limit of the testing setup.

	Measured resistance				Predicted	
	Values(N)	Avg.(N)	St. Dev.	Ratio	Value(N)	Ratio
F1 load	12.2, 10.8, 7.5, 7.9, 9.3	9.6	1.9	2.23	16.5	2.37
F2 load	3.7, 4.1, 5.0	4.3	0.7		7.0	

Table 1: Predicted and measured resistance to force applied along the negative X axis in the grasp problem in Fig. 6. Each row shows the results obtained if the preload is applied exclusively by finger 1 or finger 2 respectively. Experimental measurements were repeated 5 times for finger 1 (to account for the higher variance) and 3 times for finger 2. Predicted values are also shown both in absolute force, and ratio between the two preload cases.

7 Discussion

Limitations Our iterative approach allows us to constrain virtual object movement to the successive directions of unbalanced wrenches. However, such an iterative approach is not guaranteed to converge, or to converge to the physically meaningful state of the system. In a subset of cases, the solver reports large resistible wrenches relative to neighboring states; for example, in the grasp Case 2 from the previous section (Fig. 6), when computing resistance to disturbances sampled from the positive X half plane of the XY plane (Fig. 7), we obtain an outlier in the case of Finger 1 preload that does not follow the trend of the surrounding points. It is possible that the numerical setup of the problem for that particular direction allows the solver to "wedge" the object into the grasp, increasing resistance to the external wrench. These effects will require further

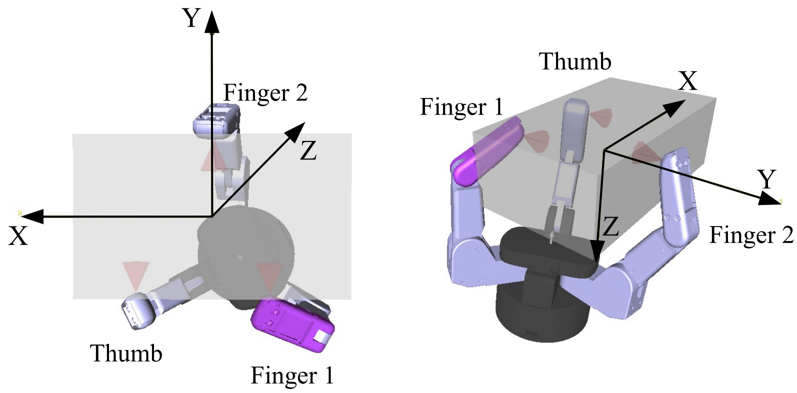


Fig. 6: Top and side views for grasp example 2 also indicating finger labels. Note that the spread angle degree of freedom of the Barrett hand changes the angle between finger 1 and finger 2; the thumb is only actuated in the flexion direction.

investigation. A promising avenue we are exploring is to also include energy constraints in the core solver, requiring that the energy used responding to an external wrench equal that provided by the wrench itself.

Furthermore, we would like to analyze the effect of uncertainties (e.g. in exact contact location) on our model. We believe exploring the sensitivity of the model to such uncertainties may yield many valuable insights.

Alternative Approaches As was described in the Problem Statement, a simpler alternative is to disregard non linear effects w.r.t. virtual object movement, i.e. assume that the joints are fixed (thus joint torque can both increase and decrease passively), and that friction forces also behave in spring-like fashion. The price for this simplicity is, that the results may not be physically sound.

At the other and of the spectrum, our iterative approach allowing successive virtual object movements in the direction of the net resultant wrench shares some of the features of a typical time-stepping dynamics engine. One could therefore forgo the quasi-static nature of our approach, assume that unbalanced wrenches produce object acceleration or impulses, and perform time integration to obtain new object poses. This approach can have additional advantages: even an unstable grasp can eventually transform into a stable one, as the object settles in the hand; a fully dynamic simulation can capture such effects. However, in highly constrained cases, such as grasps at or near equilibrium, any inaccuracy can lead to the violation of interpenetration or joint constraints, in turn requiring corrective penalty terms which add energy to the system. Our quasi-static approach only attempts to determine if an equilibrium can exist in the given state, and thus only reasons about virtual object movements, without dynamic effects.

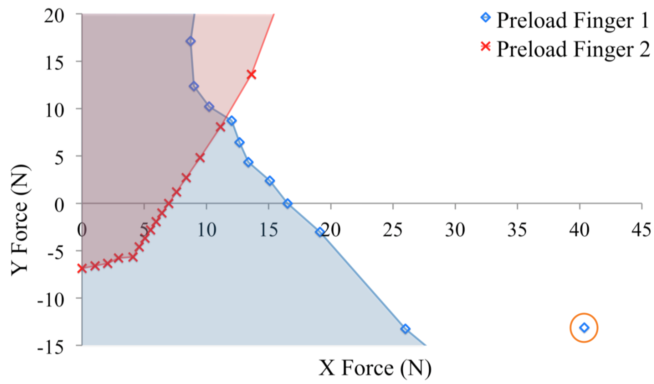


Fig. 7: Forces in an XY halfplane that can be resisted by grasp example 2 (shaded) as predicted by our framework, depending on which finger is preloaded. An outlier prediction is circled in orange.

8 Conclusions

In this paper, we have introduced an algorithm that aims to answer what we believe to be not only a meaningful theoretical question, but also one with important practical applications: *once a given joint preload has been achieved, can a grasp resist a given wrench passively, i.e. without any change in commanded joint torques?* In the inner loop of a binary search, the same algorithm allows us to determine the largest magnitude that can be resisted for a disturbance along a given direction.

In the examples above we show how the actively set joint preload combines with passive effects to provide resistance to external wrenches; our algorithm captures these effects. Furthermore, we can also compute how preloads set for some of the hand joints can cause the other joints to load as well, and the combined effects can exceed the intended or commanded torque levels. We can also study what subset of the joints is preferable to load with the purpose of resisting specific disturbances.

Our directional goal is to enable practitioners to choose grasps for a dexterous robotic hand knowing that all disturbances they expect to encounter will be resisted without further changes in the commands sent to the motors. Such a method would have wide applicability, to hands that are not equipped with tactile or proprioceptive sensors (and thus unable to sense external disturbances) and can not accurately control joint torques, but are still effective thanks to passive resistance effects.

In its current form, the algorithm introduced here can answer "point queries", for specific disturbances or disturbance directions. However, its computational demands do not allow a large number of such queries to be answered if a grasp is to be planned at human-like speeds; furthermore, the high dimensionality of the complete space of possible external wrenches generally prevents sampling approaches. GWS-based approaches efficiently compute a global measure of wrenches that can be resisted assuming perfect information and controllability of contact forces. We believe passive resistance has high practical importance for the types of hands mentioned above, but no method is currently available to efficiently distill passive resistance abilities into a single, global assessment of the grasp. We will continue to explore this problem in future work.

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